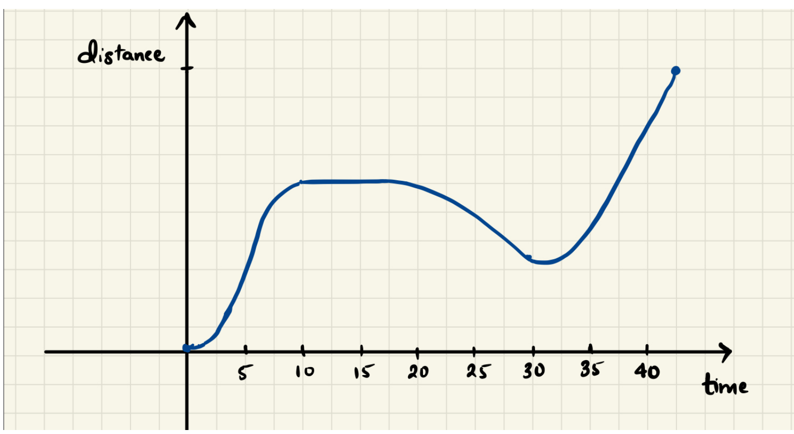
Homework 1

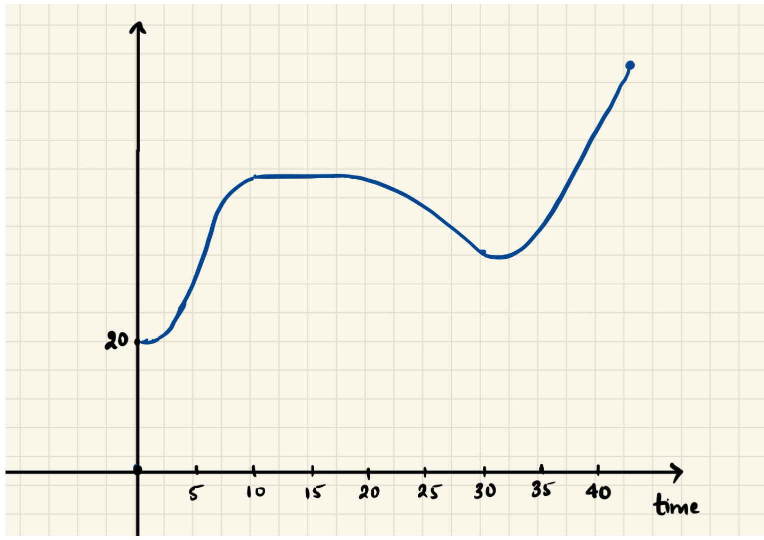
Instructions: Read the homework instructions on the syllabus. Write your solutions on paper or a writing tablet, scan it and upload it to canvas. The file must be in pdf extension. Show neat and complete work and make sure that your scan is legible. Label your solutions and make sure they are in increasing order.

1. You are traveling from Kansas City to St.Louis. You have an advanced odometer in your car that measures the distance that you are away from Kansas City. So when you start out the odometer reads 0. As you are travelling you record the odometer and see how it changes with time. You go home and you make the following plot:



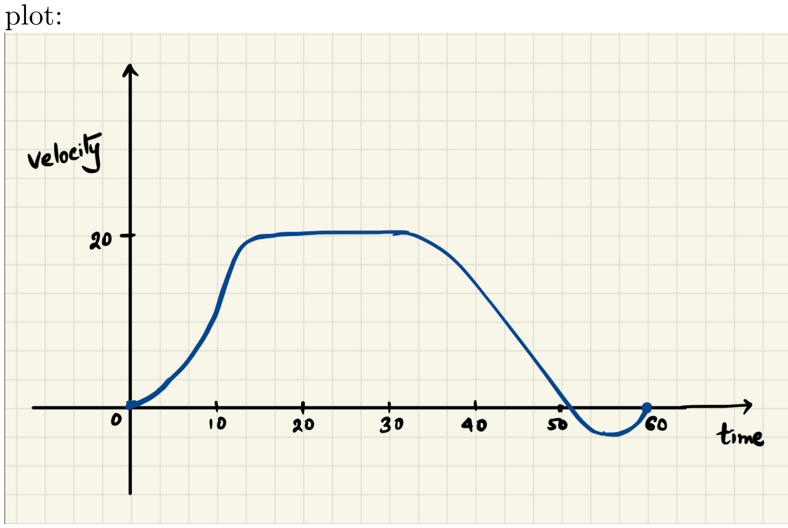
   (a) Plot the velocity vs. time graph. (You don't have to be precise with the numbers. You will be graded on the correct shape of the graph)

   (b) On a different day you get the following plot:



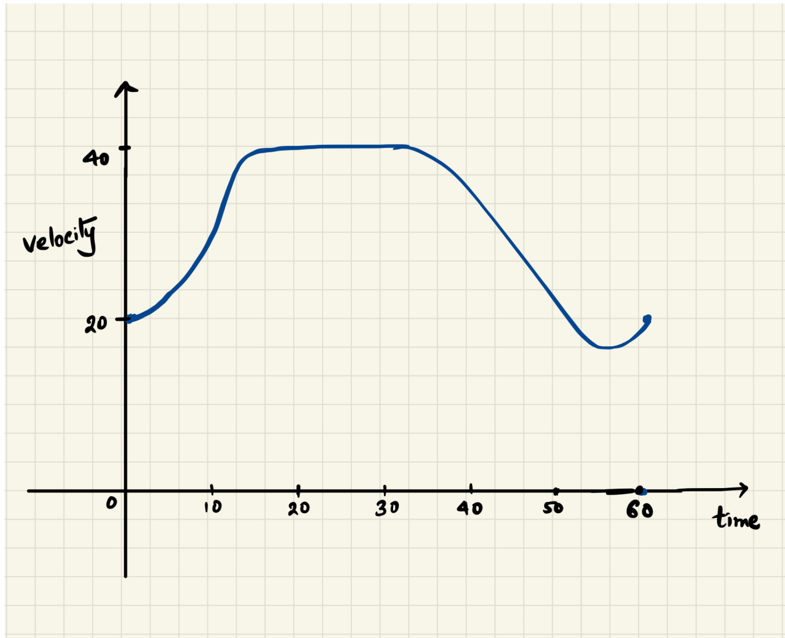
Explain the difference between the two plots in terms of distance and velocity. Will the velocity vs time graph be same or different?

2. You are again traveling from Kansas City to St.Louis. But this time you only have access to a speedometer in your car that measures your **velocity**. As you are travelling you record the speedometer and see how it changes with time. You go home and you make the following plot:



(a) Plot the distance vs time graph. (You don't have to be precise with the numerical values. Try to get the correct shape).

(b) On a different day you see the following plot:



The above plot is the plot from (a) but shifted vertically. Will the distance vs time graph be same or different?

3. Show that $x(t) = 5e^{t^2}$ is a solution to

$$x'' - x' - 2x = 10te^{t^2}(2t - 1)$$

Note that this equation is the same as

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - \frac{\mathrm{d}x}{\mathrm{d}t} - 2x = 10te^{t^2}(2t - 1)$$

4. Show that $x = e^{-2t}$ is a solution to $x'' + 4x' + 4x = 0$

5. Is $y = sin(t)$ a solution to $(\frac{\mathrm{d}y}{\mathrm{d}t})^2 = 1 - y^2$?

6. Find a solution to $(x')^2 + x^2 = 4$ using your knowledge of derivative of functions that you know from basic calculus.

7. Verify that $x = C_1e^{-t} + C_2e^{2t}$ is a solutions to $x'' - x' - 2x = 0$. Find C_1 and C_2 to solve for the initial conditions $x(0) = 10$ and $x'(0) = 0$

8. Answer the following questions for the given differential equations

- Identify the dependent variable and the independent variable.
- What is the order of the equations?
- Is it linear or non-linear?
- If it is linear, is it homogeneous or non-homogeneous? If it is non-linear write N/A for this question
- Is it an equation with constant coefficients or non-constant coefficients?

(a) $y''' + 3y + 10x = 0$

(b) $x'' + tx^2 = e^{-t}$

(c) $\frac{\mathrm{d}^4x}{\mathrm{d}t^4} = 0$

(d) $x'' + sin(x) = t^2$

(e) $y' \cdot y = y^3$
(independent variable is not clear. choose any)

(f) $\frac{\mathrm{d}^4y}{\mathrm{d}x^4} - \frac{\mathrm{d}^3y}{\mathrm{d}x^3} - e^x \cdot y = \frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(g) $x^3 + \frac{\mathrm{d}^4y}{\mathrm{d}x^4} - \frac{\mathrm{d}^3y}{\mathrm{d}x^3} - e^x \cdot y = \frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

9. Solve
$$\frac{\mathrm{d}x}{\mathrm{d}t} = t^3 + t^2 \text{ for } x(1) = 3 \tag{1}$$

10. Solve
$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4sin(4x) \text{ for } y(0) = \pi \tag{2}$$

11. Solve
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+x^2} \text{ for } y(0) = 0 \tag{3}$$

12. Solve
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+y^2} \text{ for } y(0) = 0 \tag{4}$$

13. Solve
$$\frac{\mathrm{d}x}{\mathrm{d}t} = sin(t^2) + t \text{ for } x(0) = 5 \tag{5}$$
Hint: This does not have an elementary closed form solution. It is ok to leave your answer as a definite integral.

14. Find the general solution of

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{cos(t)}{x^2} \tag{6}$$

15. Sketch the slope field for $y' = x^2$.

16. Sketch the slope field for $y' = y^2$.

17. Sketch the slope field for $y' = x(1-y)$.

18. Explain what is wrong with the following calculation:
    Find the general solution of

$$y' = y^2$$

$$\implies \frac{\mathrm{d}y}{\mathrm{d}t} = y^2$$

Integrate both sides with respect to dt

$$\implies \int \frac{\mathrm{d}y}{\mathrm{d}t} dt = \int y^2 dt$$

$$\implies \int dy = \frac{y^3}{3} + C$$

$$\implies y(t) = \frac{y^3}{3} + C$$